

STR Analysis in JMulTi

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Smooth Transition Regression models can be specified, estimated, and checked in JMulTi . Analyzing nonlinear models of the STR type with JMulTi is described in greater detail in Teräsvirta (2004). This helpsection is just a short summary for the user who already has some theoretical background.

1 The Model

The standard STR model with a logistic transition function allowed for in JMulTi has the form

$$y_t = \phi' \mathbf{z}_t + \theta' \mathbf{z}_t G(\gamma, c, s_t) + u_t, \quad u_t \sim iid(0, \sigma^2)$$
$$G(\gamma, c, s_t) = \left(1 + \exp\left\{-\gamma \prod_{k=1}^K (s_t - c_k)\right\} \right)^{-1}, \quad \gamma > 0$$

where $\mathbf{z}_t = (\mathbf{w}_t', \mathbf{x}_t)'$ is an $((m + 1) \times 1)$ vector of explanatory variables with $\mathbf{w}_t' = (1, y_{t-1}, \dots, y_{t-p})'$ and $\mathbf{x}_t' = (x_{1t}, \dots, x_{kt})'$. ϕ and θ are the parameter vectors of the linear and the nonlinear part respectively. The transition function $G(\gamma, c, s_t)$ depends on the transition variable s_t , the slope parameter γ and the vector of location parameters c . In JMulTi only the most common choices for K are available, $K = 1$ (LSTR1) and $K = 2$ (LSTR2). The transition variable s_t can be part of \mathbf{z}_t or it can be another variable, like for example $s_t = t$ (trend).

1.1 The Modelling Cycle

The modelling cycle suggested by the implementation in JMulTi consists of the three stages: specification, estimation and evaluation.

1. Specification starts with setting up a linear model that forms a starting point for the analysis. It can be modelled by using the VAR framework. The second part of specification involves testing for nonlinearity, choosing s_t and deciding whether LSTR1 or LSTR2 should be used.
2. Estimation involves finding appropriate starting values for the nonlinear estimation and estimating the model.
3. Evaluation of the model usually includes graphical checks as well as various tests for misspecification, such as error autocorrelation, parameter nonconstancy, remaining nonlinearity, ARCH and nonnormality.

2 Specifying the AR Part

The first step of the specification of an STR model is to select the linear AR model to start from. The selection mechanism allows to choose one endogenous variable y_t and an arbitrary number of exogenous (\mathbf{x}_t) and deterministic variables. The maximum lag order for y_t and \mathbf{x}_t determines the number of lags to include. Lags of y_t and \mathbf{x}_t can be excluded again via setting subset restrictions. A constant is always included, seasonal dummies can be added via a button. Because the linear part must be stationary, there is no option to include a trend.

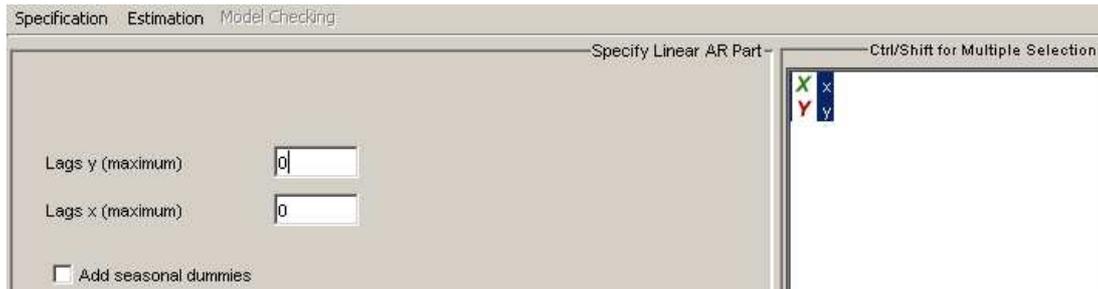


Figure 1: Specification of AR Part

2.1 Transition Variable

The transition variable s_t must be part of the selected variables or lags of these variables if it is not a trend. If s_t is not part of \mathbf{z}_t , then it can be excluded from \mathbf{z}_t via setting subset restrictions. One can also set $s_t = t$ in later stages, in that case it does not have to be included.

2.2 Smooth Trend

It is possible to specify a smooth trend model

$$y_t = \phi_0 + \theta_0 G(\gamma, c, t) + u_t$$

by just including y_t with zero lags and specifying TREND as s_t in the estimation.

2.3 Excluding Variables and Lags

The subset panel allows to impose exclusion restrictions on the selected variables by just clicking on the corresponding parameter and setting it to 0. To find those restrictions one could apply a subset search routine implemented in the VAR model part of JMULTi. The excluded variables can still be used as transition variables but they are excluded from both,

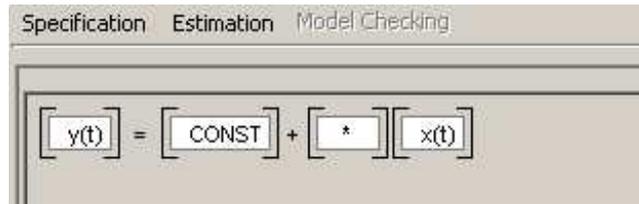


Figure 2: Subset Restrictions on the AR Part

the linear and nonlinear part, by setting $\phi_i = \theta_i = 0$. Other restrictions on the parameter space can be set in later stages of the analysis.

3 Testing Linearity

The test can be used to check, whether there is a nonlinearity of the STR type in the model. It also helps to determine the transition variable and whether LSTR1 or LSTR2 should be used. The following auxiliary regression is applied if s_t is an element of \mathbf{z}_t :

$$y_t = \beta'_0 \mathbf{z}_t + \sum_{j=1}^3 \beta'_j \tilde{\mathbf{z}}_t s_t^j + u_t^* \quad (1)$$

whith $\mathbf{z}_t = (1, \tilde{\mathbf{z}}_t)'$. In case s_t is not part of \mathbf{z}_t :

$$y_t = \beta'_0 \mathbf{z}_t + \sum_{j=1}^3 \beta'_j \mathbf{z}_t s_t^j + u_t^*$$

The null hypothesis of linearity is $H_0 : \beta_1 = \beta_2 = \beta_3 = 0$. In **JMulTi** this linear restriction is checked by applying an F test. It is denoted in the output with the F symbol.

3.1 Choosing the Transition Variable

All potential transition variables can be selected in the respective table. The test is then executed for each of the candidates and the variable with the strongest test rejection (the smallest p -value) is tagged with the symbol *. This can be used as a decision rule for choosing an appropriate s_t , especially if the differences are big.

3.2 Imposing restrictions

It is possible to carry out the linearity test under further restrictions about θ . A variable can be excluded from the nonlinear part if $\theta_i = 0$. In the linearity test this can be taken into account by setting elements of $\beta_j = 0$. In **JMulTi** this can be done by selecting elements from the respective table. If all variables are excluded from the nonlinear part, then the test cannot be computed for s_t that are part of \mathbf{z}_t . However, it still works for transition variables not contained in \mathbf{z}_t , for example t .

3.3 Numerical Problems

Because in the test regression (1) powers of s_t are included in the regressor, this might led to invertibility problems if the elements of s_t are close to zero or one. In that case the output is *NaN*.

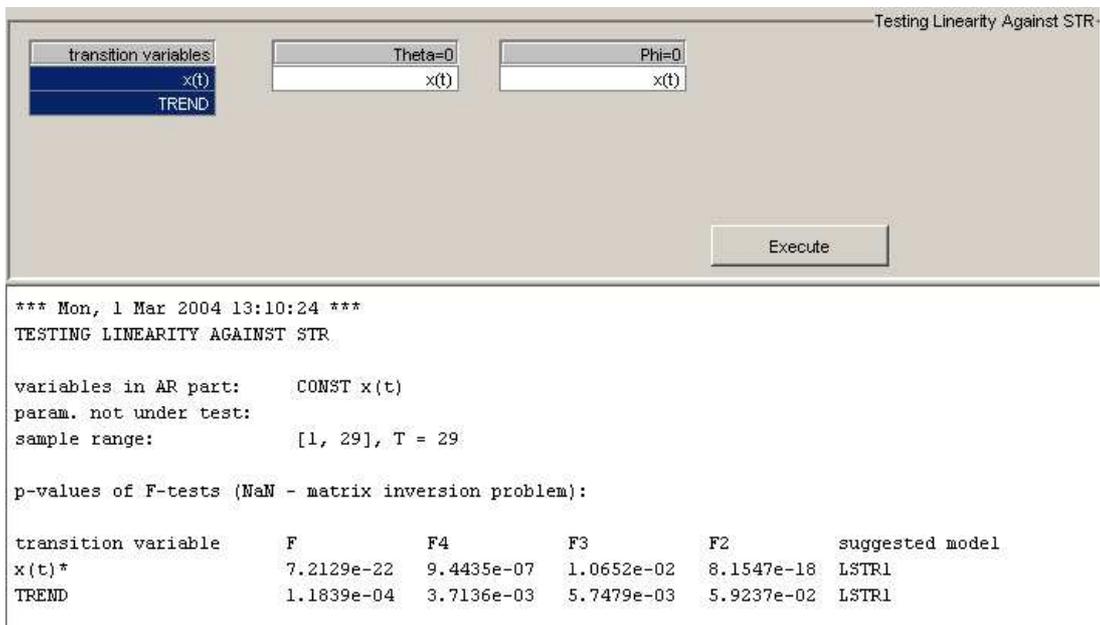


Figure 3: Testing Linearity against STR

4 Choosing the Type of Model

Once linearity has been rejected, one has to choose whether an LSTR1 or an LSTR2 model should be specified. The choice can be based on the test sequence:

1. test $H_{04} : \beta_3 = 0$
2. test $H_{03} : \beta_2 = 0 | \beta_3 = 0$
3. test $H_{02} : \beta_1 = 0 | \beta_2 = \beta_3 = 0$

The test is based on the same auxiliary regression (1) as the linearity test. In JMulTi the corresponding F-statistics of the null hypothesis H_{04}, H_{03}, H_{02} are denoted by $F4, F3, F2$. The chosen model is explicitly stated. However, if the test sequence does not provide a clear-cut choice between the alternatives, it is sensible to fit both, an LSTR1 and LSTR2 model and decide on the evaluation stage. This can be done by looking at the information criteria or at the residual sum of squares.

5 Finding Initial Values

The parameters of the STR model are estimated by a nonlinear optimization routine. It is important to use good starting values for the algorithm to work. The gridsearch creates a linear grid in c and a log-linear grid in γ . For each value of γ and c the residual sum of squares is computed. The values that correspond to the minimum of that sum are taken as starting values. It should also be noted that in order to make γ scale-free, it is divided by $\hat{\sigma}_s^K$, the K th power of the sample standard deviation of the transition variable.

5.1 Operating the Gridsearch

Selecting the transition variable For the grid search s_t has to be known already. The choice can be based on the linearity test or on a priori knowledge.

Selecting the grid Either LSTR1 or LSTR2 can be chosen. If LSTR2 is selected, then the grid is constructed over c_1, c_2, γ , whereas with LSTR1 it is over c_1, γ . The corresponding text fields can be used to adjust the number of gridpoints as well as the overall range of the parameter. In case of LSTR2, the range and grid for c_1, c_2 can only be specified jointly.

Plotting the RSS It might be helpful to plot the residual sum of squares as a function of γ and c . This can be done by selecting the plot option. There is always a surface and a contour plot. The surface plot displays -RSS, because the maximum is usually better visible in graphs of this kind. In case of LSTR2 there are 3 parameters that vary, therefore plots are conditional on distinct values of γ . The number of those plots can be adjusted.

Using the starting values The results of the gridsearch are displayed in the text field. One should check, whether there is a boundary solution, because this might indicate a problem. For convenience, the found starting values are automatically inserted in the text fields for initial values in the estimation panel.

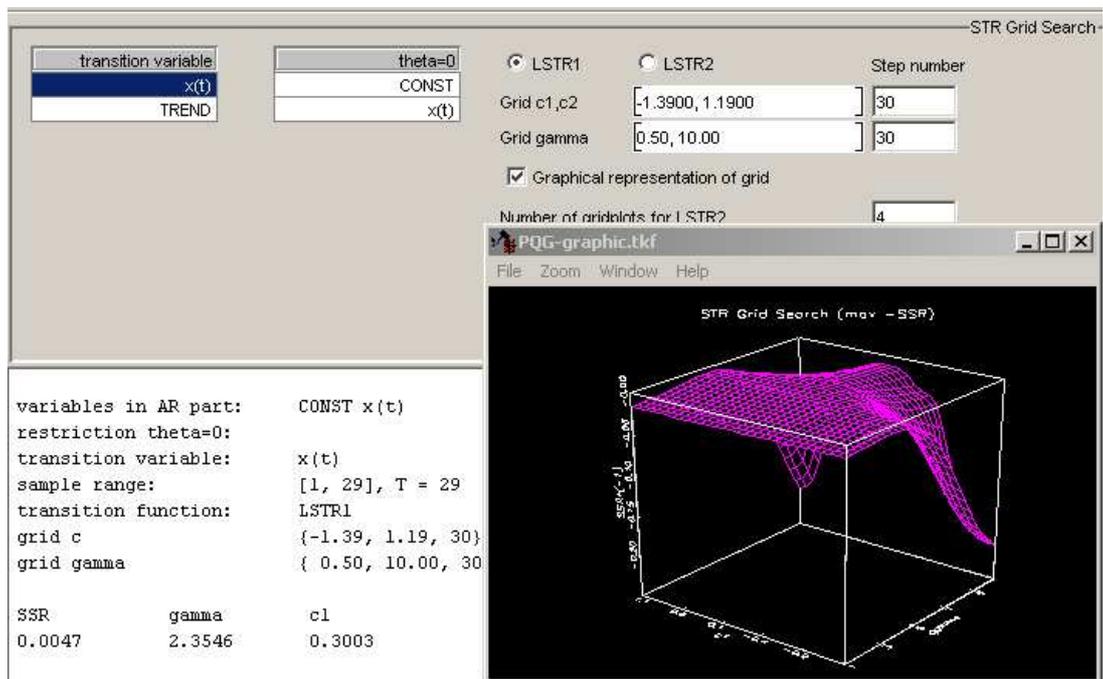


Figure 4: Grid Search to find Starting Values

6 Estimation of Parameters

Once good starting values have been found, the unknown parameters c, γ, θ, ϕ can be estimated by using a form of the Newton-Raphson algorithm to maximize the conditional maximum likelihood function.

6.1 Setting Restrictions

The following types of restrictions are available for estimation:

1. $\theta_i = -\phi_i$ the corresponding parameter will disappear if $G(\gamma, c, s_t) = 1$
2. $\phi_i = 0$ the corresponding parameter will disappear if $G(\gamma, c, s_t) = 0$
3. $\theta_i = 0$ the variable will only enter the linear part

A variable can only be selected for one of those restrictions.

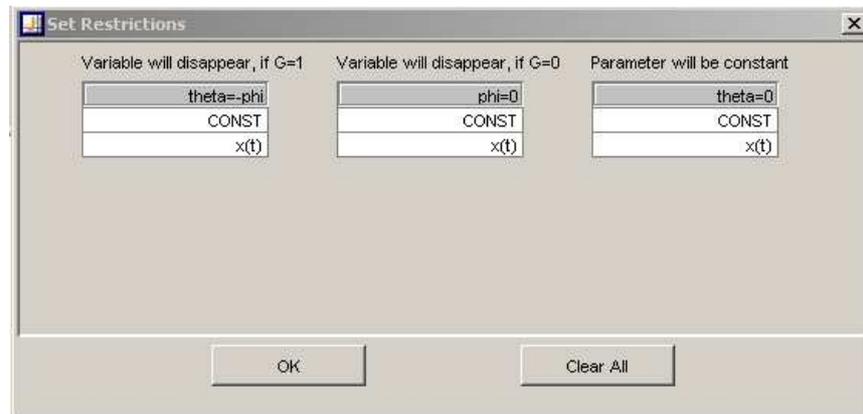


Figure 5: Setting Restrictions for Estimation

6.2 Output

The starting values, parameter estimates, standard deviations, t-values and p-values are displayed for the estimated model separated for the linear and the nonlinear part. The covariance is computed by

$$\hat{\Sigma}_{\tilde{\beta}} = 2\tilde{\sigma}^2 \left[\frac{\delta^2 S}{\delta \beta \delta \beta'} \right]^{-1} \Big|_{\tilde{\beta}}$$

The following statistics are given as well:

1. AIC, SC, HQ
2. R^2 and adjusted R^2

3. variance and standard deviation of s_t
4. variance and standard deviation of estimated residuals

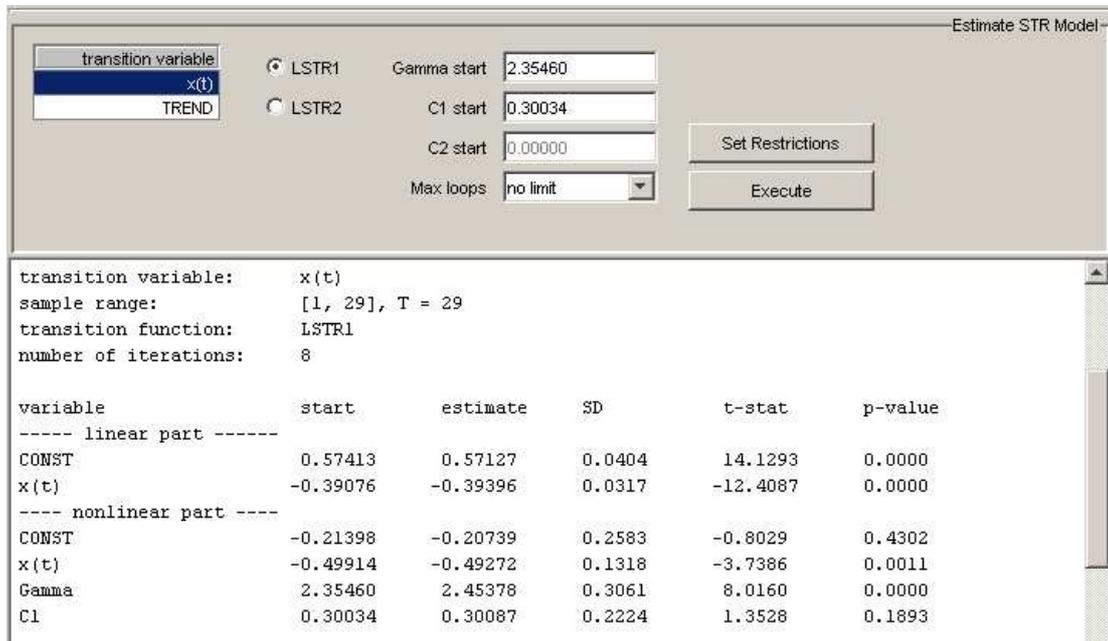


Figure 6: STR Estimation

7 Testing the STR Model

The quality of the estimated nonlinear model should be checked against misspecification like in the linear case. The tests for STR models are generalizations of the corresponding tests for misspecification in linear models.

7.1 Test of No Error Autocorrelation

The test is a special case of a general test described in Godfrey (1988) and has been discussed in its application to STR models in Teräsvirta (1998). The procedure is to regress the estimated residuals \tilde{u}_t on lagged residuals $\tilde{u}_{t-1} \dots \tilde{u}_{t-q}$ and the partial derivatives of the log-likelihood function with respect to the parameters of the model. The test statistic is then

$$F_{LM} = \{(SSR_0 - SSR_1)/q\} / \{SSR_1/(T - n - q)\}$$

where n is the number of parameters in the model, SSR_0 the sum of squared residuals of the STR model and SSR_1 the sum of squared residuals from the auxiliary regression.

7.2 Test of No Additive Nonlinearity

After the STR has been fitted, it should be checked whether there is remaining nonlinearity in the model. The test assumes that the type of the remaining nonlinearity is again of the STR type. The alternative can be defined as:

$$y_t = \phi' \mathbf{z}_t + \theta' \mathbf{z}_t G(\gamma_1, c_1, s_{1t}) + \psi' \mathbf{z}_t H(\gamma_2, c_2, s_{2t}) + u_t$$

where H is another transition function and $u_t \sim iid(0, \sigma^2)$. To test this alternative the auxiliary model:

$$y_t = \beta_0' \mathbf{z}_t + \theta' \mathbf{z}_t G(\gamma_1, c_1, s_{1t}) + \sum_{j=1}^3 \beta_j' \tilde{\mathbf{z}}_t s_{2t}^j + u_t^*$$

is used. The test is done by regressing \tilde{u}_t on $(\tilde{\mathbf{z}}_t' s_{2t}, \tilde{\mathbf{z}}_t' s_{2t}^2, \tilde{\mathbf{z}}_t' s_{2t}^3)'$ and the partial derivatives of the log-likelihood function with respect to the parameters of the model. The null hypothesis of no remaining nonlinearity is that $\beta_1 = \beta_2 = \beta_3 = 0$. The choice of s_{2t} can be a subset of available variables in \mathbf{z}_t or it can be s_{1t} . It is also possible to exclude certain variables from the second nonlinear part by restricting the corresponding parameter to zero. The resulting F statistics are given in the same way as for the test on linearity.

7.3 Test of Parameter Constancy

This is a test against the null hypothesis of constant parameters against smooth continuous change in parameters. The alternative can be written as follows:

$$y_t = \phi(t)' \mathbf{z}_t + \theta(t)' \mathbf{z}_t G(\gamma, c, s_t) + u_t, \quad u_t \sim iid(0, \sigma^2)$$

where

$$\phi(t) = \phi + \lambda_\phi H_\phi(\gamma_\phi, c_\phi, t^*)$$

and

$$\theta(t) = \theta + \lambda_\theta H_\theta(\gamma_\theta, c_\theta, t^*)$$

with $t^* = t/T$ and $u_t \sim iid(0, \sigma^2)$. The null hypothesis of no change in parameters is $\gamma_\theta = \gamma_\phi = 0$. The parameters γ and c are assumed to be constant. The following nonlinear auxiliary regression is used:

$$y_t = \beta'_0 \mathbf{z}_t + \sum_{j=1}^3 \beta'_j \mathbf{z}_t (t^*)^j + \sum_{j=1}^3 \beta'_{j+3} \mathbf{z}_t (t^*)^j G(\gamma, c, s_t) + u_t^*$$

In JMulTi the F-test results are given for three alternative transition functions

$$H(\gamma, c, t^*) = \left(1 + \exp\left\{-\gamma \prod_{k=1}^K (t^* - c_k)\right\} \right)^{-1} - \frac{1}{2}, \quad \gamma > 0$$

with $K = 1, 2, 3$ respectively and assuming that $\gamma_\theta = \gamma_\phi$.

7.4 ARCH-LM Test

The test on no ARCH is available for STR as well. It is described in the help chapter [Initial Analysis](#).

7.5 Jarque-Bera Test

The test on normality can be applied. It is described in the help chapter [Initial Analysis](#).

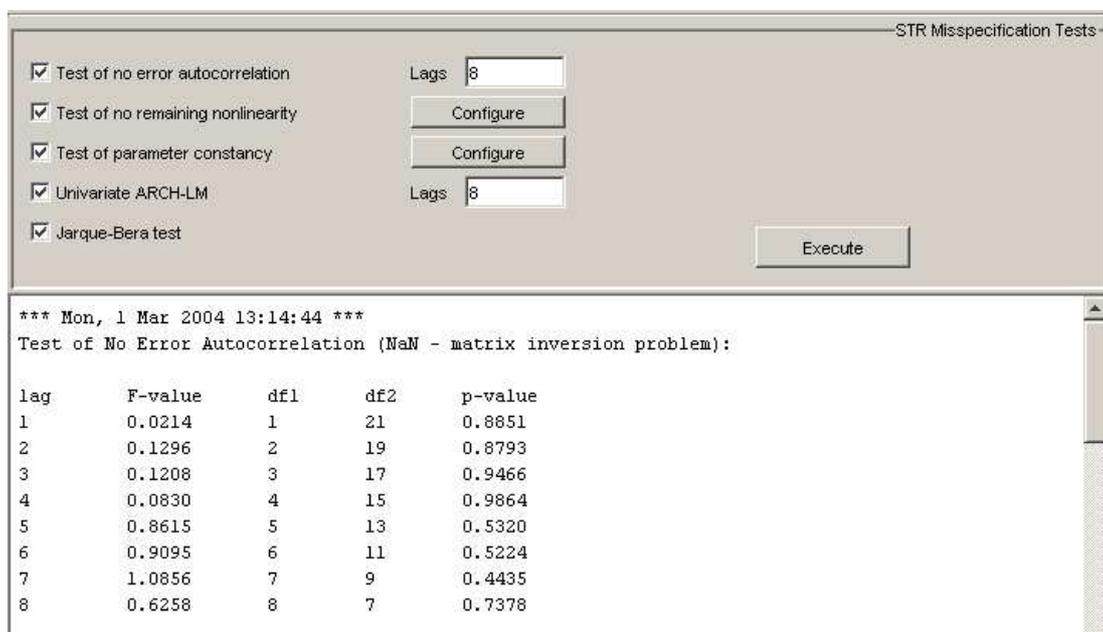


Figure 7: Diagnostic Tests for STR

8 Graphical Analysis

Various parts of the estimated STR model can easily be plotted. This can help to detect problems in the residuals quickly and to illustrate the estimation results.

8.1 Residuals

The estimated residuals

$$\tilde{u}_t = y_t - \tilde{\phi}'\mathbf{z}_t + \tilde{\theta}'\mathbf{z}_t G(\tilde{\gamma}, \tilde{c}, s_t)$$

can be plotted in various ways. It is also possible to add them to the dataset. They will appear as a new variable with the name `str_resids` in the time series list.

8.2 Plotting Parts of the Model

It is possible to plot against t :

1. the transition function $G(\tilde{\gamma}, \tilde{c}, s_t)$
2. the fitted series $\tilde{\phi}'\mathbf{z}_t + \tilde{\theta}'\mathbf{z}_t G(\tilde{\gamma}, \tilde{c}, s_t)$
3. the original series y_t
4. the nonlinear part $\tilde{\theta}'\mathbf{z}_t G(\tilde{\gamma}, \tilde{c}, s_t)$
5. the linear part $\tilde{\phi}'\mathbf{z}_t$
6. the transition variable s_t

It is also possible to plot the transition function as function of transition variable:

$$G(s_t) = \left(1 + \exp\left\{-\tilde{\gamma} \prod_{k=1}^K (s_t - \tilde{c}_k)\right\} \right)^{-1}$$

with $K = 1$ for LSTR1 or $K = 2$ for LSTR2. If **Multiple graphs** is selected, then every plot is shown in a separate diagram. The diagrams are arranged arraywise in one window. If this option is not selected, all plots appear in just one diagram.

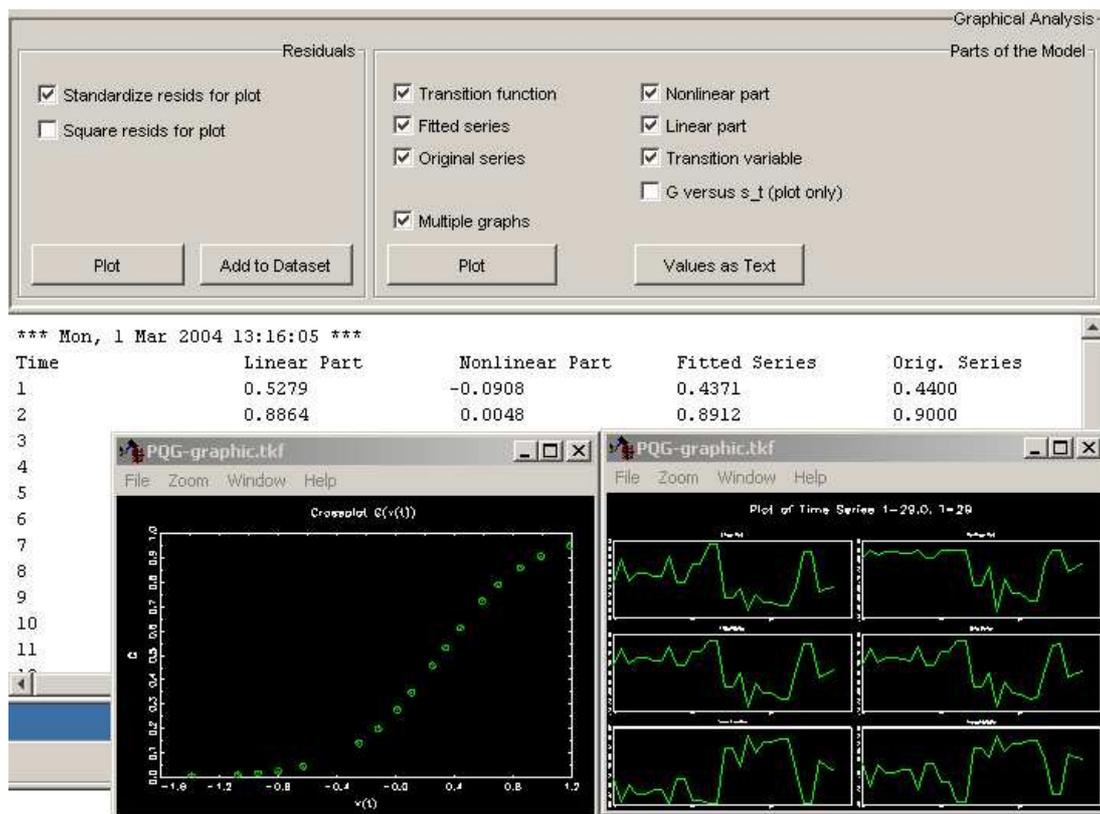


Figure 8: Graphical Analysis

References

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- Teräsvirta, T. (1998). Modeling economic relationships with smooth transition regressions, *in* A. Ullah and D. Giles (eds), *Handbook of Applied Economic Statistics*, Dekker, New York, pp. 229–246.
- Teräsvirta, T. (2004). Smooth transition regression modelling, *in* H. Lütkepohl and M. Kräzig (eds), *Applied Time Series Econometrics*, Cambridge University Press, Cambridge.